RELATIONSHIPS BETWEEN ABSORPTION, BACKSCATTER AND LIQUID WATER CONTENT OF THE MAJOR CLOUD TYPES

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INTRODUCTION

Both fog and cloud have an important effect on radiation transfer in the atmosphere. Recently Chylek (1979) and Pinnick et al. (1979) have shown that in the case of atmospheric fog, at particular infrared wavelengths there do exist approximate linear relationships, that are independent of the form of the size distribution, between extinction, absorption and liquid water content (LWC). In this paper we extend these relationships between extinction, absorption and LWC to water droplet clouds which cover the major cloud categories.

We show that for visible and near-infrared wavelengths the cloud extinction coefficient $\sigma_e (km^{-1})$ is uniquely related to the backscatter coefficient $\sigma_b (km^{-1}str^{-1})$, independent of the form of the cloud drop size distribution. At the ruby laser wavelength $\lambda=0.694\mu m$ the relation is $\sigma_e=16 \sigma_b$. We also show however, that cloud liquid water content for clouds of unknown drop size distribution cannot be inferred from visible, infrared or near-infrared backscatter measurements alone.

EXTINCTION, ABSORPTION, BACKSCATTER AND LIQUID WATER CONTENT OF WATER CLOUDS

Consider a polydispersion of spherical droplets characterized by a size distribution $n(r)$ and refractive index $m$. We want to investigate relationships between the cloud extinction and absorption coefficients $\sigma_e$ and $\sigma_a$, the backscatter coefficient $\sigma_b$, and the cloud liquid water content $W$ given by

$$\sigma_e = 4\pi r^2 Q_e(m,x)n(r)dr$$

$$\sigma_a = 4\pi r^2 Q_a(m,x)n(r)dr$$

$$\sigma_b = \frac{4}{3}\pi r^3 G(m,x)n(r)dr$$

$$W = \rho_d \frac{4}{3}\pi r^3 n(r)dr$$

where $\rho$ is the water droplet density, $Q_e(m,x)$ and $Q_a(m,x)$ are the efficiency factors for extinction and absorption for a droplet with refractive index $m$ and size parameter $x=2\pi r/\lambda$, and $G(m,x)$ is the backscatter efficiency (or gain) defined as the ratio of the backscatter cross section to the geometric area.

The efficiency factors $Q_e(x)$ and $Q_a(x)$ for droplets having size parameter $x \leq x_m (x_m = 2\pi r_m/\lambda)$ can be approximated by linear functions of droplet size parameter $Q_e(x,\lambda) = c(\lambda)x$ and $Q_a(x,\lambda) = c'(\lambda)x$ as shown by Chylek (1979) and Pinnick et al. (1979).

The consequence of utilizing these simple linear approximations for the Mie efficiency factors in the expressions for the cloud extinction and absorption, given by Eqs. 1, 2 are far reaching. This is because these expressions now contain the integral $j n(r)dr$ and thus the coefficients become proportional to cloud water content $W$ and independent of the particle size distribution $n(r)$:

$$\sigma_e = \frac{3\pi c}{2\lambda \rho} W$$

$$\sigma_a = \frac{3\pi c'}{2\lambda \rho} W$$

where $c(\lambda)$, $c'(\lambda)$ are the slopes of the straight lines approximating the Mie efficiency curves.

SELECTED CLOUD SIZE DISTRIBUTIONS

The cloud droplet size measurements used here we judge to be fairly reliable and were chosen to represent adequately the major cloud types ranging from cumulus, continental and maritime cumulus [Dien (1948), Battan & Reitan (1957), Squires (1958), Durbin (1959), Jiusto (1967), Warner (1969, 1973a, b), Fitzgerald (1972), Fitzgerald & Sprye-Duran (1973), Ryan et al. (1972) and Eagan et al. (1974)], stratus and strato-cumulus [Dien (1948), Singleton & Smith (1960), Jiusto (1967), Sprye-Duran (1972), Fitzgerald & Sprye-Duran (1973) Ryan et al. (1972) and Eagan et al. (1974)] orographic [Squires (1955)] and cumulus congestus, cumulonimbus [Dien (1948), auffn Kempe & Weickmann (1952), Battan & Reitan (1957)]. The main sampling technique employed to obtain the cloud droplet size distributions was that of impaction onto coated slides or applicator whose collection efficiencies were corrected. The practical lower limit for detection of cloud droplets by the impaction technique is around 1.5 $\mu m$ radius. The sole cloud size determination by a light scattering counter (Ryan et al. 1972) was calibrated by means of uniformly sized water droplets.

Altogether, 158 different size distributions were used in the analysis making use of the originally measured size categories which were digitised accordingly. Only non-precipitating clouds were used in the analysis and measurements which showed evidence of glaciation were excluded.
Using a Mie scattering programme and index of refraction of water as given by Hale & Query (1973) and Ray (1972) we have calculated $\sigma_e$, $\sigma_a$, $\sigma_s$ and $\lambda$ given by eqns. (1) to (4) for the 158 cloud size distributions $n(r)$ at wavelengths $\lambda = 0.55, 0.694, 1.06, 3.8, 10.6, 1363, 2142.9$ and 3194.5 $\mu$m.

EXTINCTION, ABSORPTION AND LIQUID WATER CONTENT IN CLOUDS

The result for the volume extinction coefficient $\sigma_e$ vs $\lambda$ at $\lambda=10.6 \mu$m together with the approximation (5) is shown in Fig.1. At $\lambda=10.6 \mu$m the $Q_0=\infty$ approximation is a good one (within a factor 2) for all size distributions except those with a large range of droplet size, such as cumulus congestus, cumulonimbus, and some layer clouds. This is in good agreement with the prediction of Chylek (1978) and Pinnick et al (1979) that the $Q_0=\infty$ approximation is valid except for those size distributions which contain a large number of droplets with radii $> 14 \mu$m. The numerical results at visible and near infrared wavelengths showed that no relation between extinction and LWC independent of size distribution exists - since the $Q_0=\infty$ approximation is valid only for water droplets $< 1.0 \mu$m in this wavelength region (Chylek, 1978).

that droplets with maximum radii of $14 \mu$m is adequate for the verification of $Q_0=\infty$ for water droplets at $\lambda=3.8 \mu$m (Pinnick et al 1979).

BACKSCATTER AND EXTINCTION IN CLOUDS

For a realistic size distribution of cloud droplets we can expect to have a fairly uniform distribution of droplets throughout small ranges of drop sizes, let us say $\Delta x=1 \mu$m (corresponding to $\Delta x=10$ at $\lambda=0.694 \mu$m). Under this rather minor constraint we then average the exact Mie values $G(x)$ over intervals $\Delta x=10$. These averaged values $G(x)$ (Fig. 2) are nearly constant (except for the smallest drops) for all realistic cloud drop sizes. Thus to first order we can replace $G(x)$ in Eq.3 by a constant value $G(x,\lambda)=g(\lambda)$ that is independent of size parameter and depends only on the radiation wavelength $\lambda$: the extinction in cloud is dominated by droplets with radii $2 \mu m < x < 90 \mu m$ and so the extinction efficiency in Eq.1 can justifiably be approximated by $Q_0=1$. This approximation together with the approximation for backscatter gain $G=g(\lambda)$ leads to the cloud extinction coefficient being linearly related to the backscatter coefficient

$$\sigma_e = \frac{6n}{g(\lambda)} \sigma_{bs}$$  \hspace{1cm} (7)

Fig.1 Variation of extinction coefficient with liquid water content in cloud for 158 size distribution measurements, covering all the major cloud types. In the infrared spectral region around $\lambda=10.6 \mu$m, there exists a linear, size-distribution-independent relation between the volume extinction coefficient $\sigma_e (km^{-1})$ and the liquid water content $W(g m^{-3})$ of the form of Eq.(5), shown by the straight line.

A linear relation between cloud absorption and LWC according to (6) was found to be within a factor 2 for all cloud types at $\lambda=3.8 \mu$m. This is in general accordance with the requirement

Fig.2 The averaged backscatter gain $G(x,\lambda)$ for water droplets at a wavelength $\lambda=0.694 \mu$m (refractive index $m=1.331 - 3.35 \times 10^{-6}$). $G(x,\lambda)$ is averaged over size parameter $\Delta x=10$ (and intervals $\Delta x=20$ for $x\geq 150$) and can be approximated by a constant value $G(x,\lambda)=g(\lambda)$.

where $g(\lambda)$ is determined by numerically averaging the values of $G(x)$ as in Fig. 2. (The resulting averages are $g(\lambda=0.694 \mu m)=1.52$, $g(\lambda=1.06 \mu m)=1.50$).

To test the validity of the extinction-backscatter relation (7), we calculated using Mie theory the extinction coefficient according to Eq.1 and the backscatter coefficient according to Eq. 3 for the 158 different cloud droplet size distributions. Plotted in Fig.3 for each cloud size distribution are values of the extinction coefficient as a function of the backscatter coefficient at $\lambda=0.694 \mu m$. The linear relation between extinction and backscatter co-
We already know the backscatter gain (averaged over about 1 μm radius intervals) at a wavelength \( \lambda = 0.694 \mu m \) is nearly constant \( G(x) \approx 8 \) and not proportional to the particle size parameter (Fig. 2). Hence there can be no size distribution-independent relation between LWC and backscatter coefficient at this wavelength as the ratio of these quantities contains the ratio of the third to second moments of the droplet size distribution. To obtain a quantitative measure of this size distribution dependence, we performed Mie calculations of the backscatter coefficient using Eq. 3 and LWC using Eq. 4 for the 158 cloud size distributions. The results of these calculations at \( \lambda = 0.694 \mu m \) are presented in Fig. 4 and show that for a particular backscatter coefficient the cloud liquid water content can vary by as much as an order of magnitude with the droplet size distribution.

Fig. 4 Volume backscatter coefficient at \( \lambda = 0.694 \mu m \) versus cloud liquid water content for 158 measured droplet size distributions of cumulus and stratus type clouds. The results show that cloud liquid water content is not uniquely related to the backscatter coefficient.

Similar investigations of a possible relation between cloud LWC and backscatter coefficient at other infrared, visible, and near-infrared wavelengths \( \lambda = 0.95 \mu m, 1.06 \mu m, 3.8 \mu m, 10.6 \mu m, 1364 \mu m, 2143 \mu m \) and 3192 \( \mu m \) show again that no unambiguous relations exist, and further that for a fixed backscatter coefficient
At these other wavelengths the cloud liquid water content is generally an even more sensitive function of the droplet size distribution.

CONCLUSIONS

For all types of clouds consisting of spherical water droplets a linear relation between their extinction and backscatter coefficients at visible and near-infrared wavelengths has been derived. The relation is independent of the cloud size distribution. The relation should enable the determination of cloud extinction coefficient solely from a lidar return, providing the contribution of multiply-scattered photons to the lidar return can be neglected. However, no size-distribution-independent relation exists between cloud water content and backscatter coefficient at visible, infrared or nearmillimeter wavelengths.

The prediction of Chylek (1978) of a linear relation, independent of the size distribution between extinction at λ around 10.6 μm and liquid water content of cloud has been verified to within a factor 2 for all cloud size distributions, the exceptions being cloud types possessing a large range of drop sizes (cumulus congestus, cumulonimbus and some layer clouds). Integrated liquid water content along a path in cloud could be inferred from a CO₂ laser (λ=10.6 μm) transmissometer measurement according to (5). A similar linear relation between cloud droplet absorption at λ=3.8 μm and liquid water content has also been validated (within a factor 2) for all cloud types. The relation between cloud absorption and liquid water content can also be used, from a knowledge of cloud water content, to calculate cloud emissivity.

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